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Correction to “Packetized Predictive Control of Stochastic Systems over Bit-Rate Limited Channels with Packet Loss”

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Abstract—We correct the results in Section V of the above mentioned manuscript.

In [1], we showed that a particular class of networked control system (NCS) with quantization, i.i.d. dropouts and disturbances can be described as a Markov jump linear system of the form

$$\theta_{k+1} = \bar{A}(d_k)\theta_k + \bar{B}(d_k)\nu_k, \quad (1)$$

where

$$\theta_k \triangleq \begin{bmatrix} x_k \\ b_{k-1} \end{bmatrix} \in \mathbb{R}^{n+N}, \quad \nu_k \triangleq \begin{bmatrix} w_k \\ n_k \end{bmatrix} \in \mathbb{R}^{m+N}$$

and $\{d_k\}_{k \in \mathbb{N}_0}$ is a Bernoulli dropout process, with

$$\text{Prob}(d_k = 1) = p \in (0, 1).$$

Throughout [1] we showed that properties of the NCS can be conveniently stated in terms of the expected system matrices

$$\begin{aligned} \mathcal{A}(p) &\triangleq \mathbb{E}\{\bar{A}(d_k)\} \\ \mathcal{B}(p) &\triangleq \mathbb{E}\{\bar{B}(d_k)\} = [\mathcal{B}_w \quad \mathcal{B}_n(p)] \end{aligned}$$

and the matrix $\tilde{\mathcal{A}} = \bar{A}(1) - \bar{A}(0)$. Unfortunately, Theorem 4 in Section V-A of [1] is incorrect. For white disturbances $\{w_k\}_{k \in \mathbb{N}_0}$, the statement should be as given below. Non-white $\{w_k\}_{k \in \mathbb{N}_0}$ can be accommodated by using standard state augmentation techniques; see, e.g., [2].

Theorem 4: Suppose that (1) is MSS and AWSS and that $\{w_k\}_{k \in \mathbb{N}_0}$ is white with $\sigma_w^2 = \text{tr} R_w(0)$. Define

$$\begin{aligned} \mathcal{F}(z) &\triangleq (zI - \mathcal{A}(p))^{-1} \\ \mathcal{C}(p) &\triangleq (\sigma_w^2/m)\mathcal{B}_w\mathcal{B}_w^T + (\sigma_n^2/N)(1-p)\mathcal{E} \in \mathbb{R}^{(n+N) \times (n+N)}, \end{aligned} \quad (2)$$

where (see [1, Sec.2] for definitions)

$$\mathcal{E} \triangleq \frac{\mathcal{B}_n(p)\mathcal{B}_n(p)^T}{(1-p)^2} = \begin{bmatrix} B_1 e_1^T (\Psi^T \Psi)^{-1} e_1 B_1^T & B_1 e_1^T (\Psi^T \Psi)^{-1} \\ (\Psi^T \Psi)^{-1} e_1 B_1^T & (\Psi^T \Psi)^{-1} \end{bmatrix}. \quad (3)$$

Then, the spectral density of $\{\theta_k\}_{k \in \mathbb{N}_0}$ is given by

$$S_\theta(e^{j\omega}) = \mathcal{F}(e^{j\omega})(p(1-p)\tilde{\mathcal{A}}R_\theta(0)\tilde{\mathcal{A}}^T + \mathcal{C}(p))\mathcal{F}^T(e^{-j\omega}), \quad (4)$$

where $R_\theta(0)$ solves the following linear matrix equation:

$$R_\theta(0) = \mathcal{A}(p)R_\theta(0)\mathcal{A}(p)^T + p(1-p)\tilde{\mathcal{A}}R_\theta(0)\tilde{\mathcal{A}}^T + \mathcal{C}(p). \quad (5)$$

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Proof: See the appendix. ■

To further elucidate the situation, we note that (5) is linear and that its solution can be stated as the linear combination

$$R_\theta(0) = (\sigma_w^2/m)R_\theta^w(0) + (\sigma_n^2/N)R_\theta^n(0), \quad (6)$$

where $R_\theta^w(0)$ and $R_\theta^n(0)$ satisfy

$$\begin{aligned} R_\theta^w(0) &= \mathcal{A}(p)R_\theta^w(0)\mathcal{A}(p)^T + p(1-p)\tilde{\mathcal{A}}R_\theta^w(0)\tilde{\mathcal{A}}^T + \mathcal{B}_w\mathcal{B}_w^T \\ R_\theta^n(0) &= \mathcal{A}(p)R_\theta^n(0)\mathcal{A}(p)^T + p(1-p)\tilde{\mathcal{A}}R_\theta^n(0)\tilde{\mathcal{A}}^T + (1-p)\mathcal{E}. \end{aligned}$$

Therefore, the distortion D defined by (52) in [1] is given by

$$D \triangleq \text{tr}(\tilde{Q}R_\theta(0)) + \lambda[0 \quad e_1^T]R_\theta(0)[0 \quad e_1^T]^T,$$

where \tilde{Q} is given in terms of the Kronecker product

$$\tilde{Q} \triangleq \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \otimes Q.$$

Thus, $D = \alpha\sigma_n^2 + \beta$, with

$$\begin{aligned} \alpha &= (1/N)\text{tr}(\tilde{Q}R_\theta^n(0)) + (\lambda/N)[0 \quad e_1^T]R_\theta^n(0)[0 \quad e_1^T]^T \\ \beta &= (\sigma_w^2/m)\text{tr}(\tilde{Q}R_\theta^w(0)) + (\lambda\sigma_w^2/m)[0 \quad e_1^T]R_\theta^w(0)[0 \quad e_1^T]^T. \end{aligned}$$

The above expressions replace Lemma 11 of [1].

To derive a noise-shaping model, (6) can be substituted into (4) to provide

$$S_\theta(e^{j\omega}) = \mathcal{F}(e^{j\omega})((\sigma_w^2/m)\mathcal{K}_w\mathcal{K}_w^T + (\sigma_n^2/N)\mathcal{K}_n\mathcal{K}_n^T)\mathcal{F}^T(e^{-j\omega}),$$

where \mathcal{K}_w and \mathcal{K}_n are obtained from the factorizations

$$\begin{aligned} \mathcal{K}_w\mathcal{K}_w^T &= \mathcal{B}_w\mathcal{B}_w^T + p(1-p)\tilde{\mathcal{A}}R_\theta^w(0)\tilde{\mathcal{A}}^T \\ \mathcal{K}_n\mathcal{K}_n^T &= (1-p)(\mathcal{E} + p\tilde{\mathcal{A}}R_\theta^n(0)\tilde{\mathcal{A}}^T). \end{aligned}$$

If we define

$$\mathcal{H}(z) \triangleq [I \quad 0] \mathcal{F}(z),$$

then the above provides the noise-shaping model depicted in Fig. 2. The latter replaces Fig. 2 and Corollary 1 of [1].

Remark 1: We would like to emphasize that Theorem 4 can also be proven by adapting results in [3]–[5]. However, the noise shaping interpretation in Fig. 2 does not explicitly need an additional noise term to quantify second-order dropout effects, as opposed to what is done in [3]–[5]. □

The upper bound on the coding rate provided by Theorem 5 in [1] is also no longer correct, since it relied upon $R_\theta(0)$. The new Theorem 5 is provided below:

Theorem 5: For any $1 \leq N \in \mathbb{N}$, the minimum bit-rate R of \vec{u}_k satisfies:

$$R(D) \leq \frac{1}{2} \log_2 (\det(I + (N/\sigma_n^2)R_\xi(0))) + \frac{N}{2} \log_2 \left(\frac{\pi e}{6} \right) + 1, \quad (7)$$

where

$$R_\xi(0) = [\Gamma \quad 0] R_\theta(0) [\Gamma \quad 0]^T.$$

Proof: Follows immediately from (73) in [1] by omitting the last step where $R_\xi(0)$ was written in terms of $R_x(0)$ and (50) was used. ■

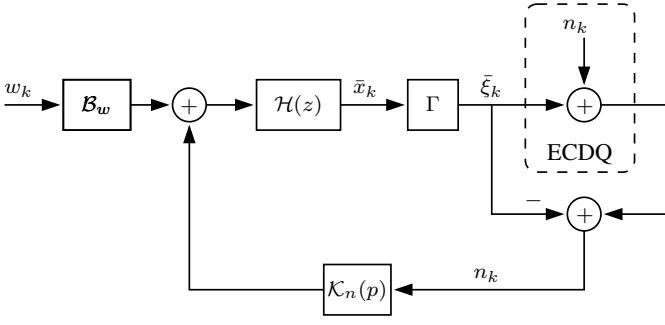


Fig. 2. Noise-Shaping Model of the NCS

Note that, in view of (6), the bound in (7) provides

$$\lim_{\sigma_n^2 \rightarrow \infty} R(D) \leq \frac{1}{2} \log_2 (\det(I + [\Gamma \ 0] R_\theta^n(0) [\Gamma \ 0]^T)) + \frac{N}{2} \log_2 \left(\frac{\pi e}{6} \right) + 1,$$

expression, which is positively bounded away from zero and replaces (58) in [1].

Remark 2: By using results in [6, Sec.5], the covariance matrix $R_\theta(0)$ can be expressed explicitly in terms of Kronecker products and matrix inversions. Specifically, let

$$G \triangleq \mathcal{A}(p) \otimes \mathcal{A}(p)^T + p(1-p) \tilde{\mathcal{A}} \otimes \tilde{\mathcal{A}}^T$$

and let $c \in \mathbb{R}^{(n+N)^2}$ be the vectorized version of the matrix $\mathcal{C}(p)$ given in (2). Then, the vectorized version of $R_\theta(0)$ is simply given by $r = (I - G)^{-1}c$. Using this approach, it is straight-forward to numerically evaluate the rate and distortion in (7). \square

We finalize this note by revisiting the NCS considered in Section V-C of [1]. Fig. 3 illustrates the rate and distortion trade-off for different horizon lengths and a fixed packet loss probability $p = 0.0085$. It may be noticed that the distortion can be reduced by using a longer horizon length in addition to increasing the bit-rate. Fig. 4 shows that when the packet-loss probability increases, it is necessary to use a larger horizon length to guarantee stability and thereby reduce the distortion.

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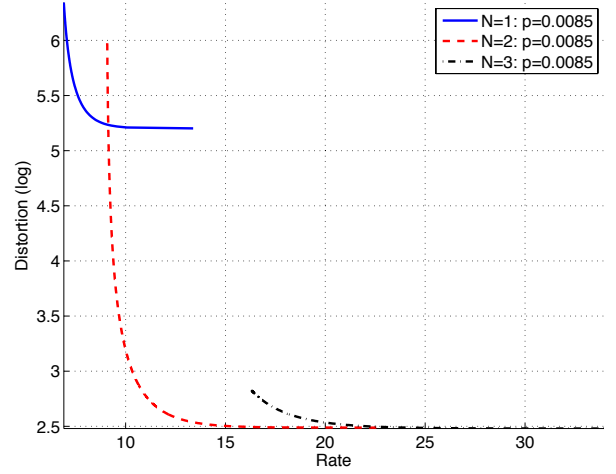


Fig. 3. Bound on $D(R)$ obtained from (7) for a fixed $p = 0.0085$ and different horizon lengths $N = 1, 2, 3$. The distortion is here expressed in the log-domain.

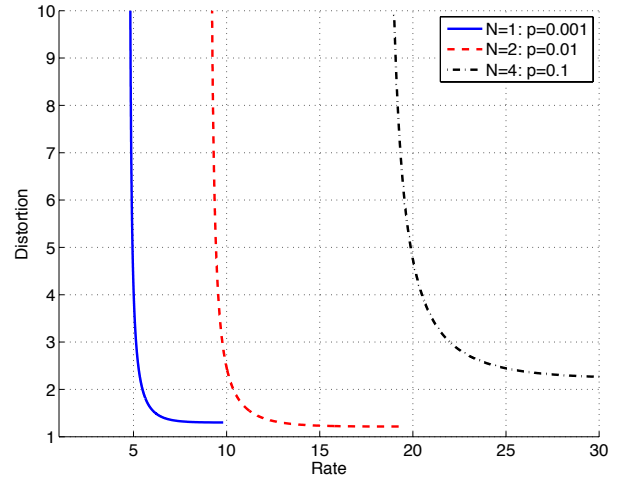


Fig. 4. Bound on $D(R)$ obtained from (7) for different packet loss probabilities and different horizon lengths.

APPENDIX PROOF OF THEOREM 4

Since $\{\nu_k\}_{k \in \mathbb{N}_0}$ is white and thus $\mathbb{E}\{\theta_k \nu_k^T\} = 0$, the system recursion (1) provides

$$\mathbb{E}\{\theta_{k+1} \theta_{k+1}^T\} = \mathbb{E}\{\bar{A}(d_k) \theta_k \theta_k^T \bar{A}(d_k)^T\} + \mathbb{E}\{\bar{B}(d_k) \nu_k \nu_k^T \bar{B}(d_k)^T\}.$$

Therefore, by conditioning on d_k and using the law of total expectation, we obtain:

$$\begin{aligned} \mathbb{E}\{\theta_{k+1} \theta_{k+1}^T\} &= p \mathbb{E}\{\bar{A}(d_k) \theta_k \theta_k^T \bar{A}(d_k)^T \mid d_k = 1\} \\ &\quad + (1-p) \mathbb{E}\{\bar{A}(d_k) \theta_k \theta_k^T \bar{A}(d_k)^T \mid d_k = 0\} \\ &\quad + p \mathbb{E}\{\bar{B}(d_k) \nu_k \nu_k^T \bar{B}(d_k)^T \mid d_k = 1\} \\ &\quad + (1-p) \mathbb{E}\{\bar{B}(d_k) \nu_k \nu_k^T \bar{B}(d_k)^T \mid d_k = 0\} \\ &= p \bar{A}(1) \mathbb{E}\{\theta_k \theta_k^T\} \bar{A}(1)^T + (1-p) \bar{A}(0) \mathbb{E}\{\theta_k \theta_k^T\} \bar{A}(0)^T \\ &\quad + p \bar{B}(1) R_\nu(0) \bar{B}(1)^T + (1-p) \bar{B}(0) R_\nu(0) \bar{B}(0)^T, \end{aligned}$$

where we have used the fact that $\{d_k\}_{k \in \mathbb{N}_0}$ is Bernoulli and ν_k and θ_k are independent of d_k . Direct algebraic manipulations allow us to

rewrite the above as

$$\mathbb{E}\{\theta_{k+1}\theta_{k+1}^T\} = \mathcal{A}(p)\mathbb{E}\{\theta_k\theta_k^T\}\mathcal{A}(p)^T + p(1-p)\tilde{\mathcal{A}}\mathbb{E}\{\theta_k\theta_k^T\}\tilde{\mathcal{A}}^T + \mathcal{C}(p). \quad (8)$$

In a similar way, one can derive that

$$\begin{aligned} \mathbb{E}\{\theta_{k+\ell+1}\theta_k^T\} &= \mathbb{E}\{(\bar{A}(d_{k+\ell})\theta_{k+\ell} + \bar{B}(d_{k+\ell})\nu_{k+\ell})\theta_k^T\} \\ &= \mathbb{E}\{\bar{A}(d_{k+\ell})\theta_{k+\ell}\theta_k^T\} + \mathbb{E}\{\bar{B}(d_{k+\ell})\nu_{k+\ell}\theta_k^T\} \\ &= \mathcal{A}(p)\mathbb{E}\{\theta_{k+\ell}\theta_k^T\} + \mathcal{B}(p)\mathbb{E}\{\nu_{k+\ell}\theta_k^T\} \\ &= \mathcal{A}(p)\mathbb{E}\{\theta_{k+\ell}\theta_k^T\}, \quad \forall \ell \in \mathbb{N}_0, \end{aligned} \quad (9)$$

since $\{\nu_k\}_{k \in \mathbb{N}_0}$ is white and θ_k and $\theta_{k+\ell}$ are independent of $d_{k+\ell}$ for non-negative values of ℓ . Equation (9) gives the explicit expression

$$\mathbb{E}\{\theta_{k+\ell}\theta_k^T\} = \mathcal{A}(p)^\ell \mathbb{E}\{\theta_k\theta_k^T\}, \quad \forall \ell \in \mathbb{N}_0. \quad (10)$$

Since the system is AWSS, we have $\lim_{k \rightarrow \infty} \mathbb{E}\{\theta_{k+1}\theta_{k+1}^T\} = R_\theta(0)$, the stationary covariance matrix of $\{\theta_k\}_{k \in \mathbb{N}_0}$. By (8) and results in [7], [8], the latter is given by the solution to (5).

On the other hand, in steady state, (10) gives that the covariance function

$$R_\theta(\ell) = \mathcal{A}(p)^\ell R_\theta(0), \quad \forall \ell \in \mathbb{N}_0. \quad (11)$$

Consequently, the positive real part of the spectrum of $\{\theta_k\}_{k \in \mathbb{N}_0}$ is given by

$$\begin{aligned} S_\theta^+(z) &= \frac{1}{2}R_\theta(0) + \sum_{\ell=1}^{\infty} R_\theta(\ell)z^{-\ell} \\ &= ((1/2)I + \mathcal{A}(p)(zI - \mathcal{A}(p))^{-1})R_\theta(0), \end{aligned}$$

where we have used the fact that, by assumption, (1) is MSS and AWSS, thus $\mathcal{A}(p)$ is Schur (see Lemma 4 in [1]) and the geometric

series

$$\sum_{n=0}^{\infty} (\mathcal{A}(p)z^{-1})^n = (I - \mathcal{A}(p)z^{-1})^{-1}.$$

Since $\{\theta_k\}_{k \in \mathbb{N}_0}$ is AWSS, its spectrum satisfies [9]

$$\begin{aligned} S_\theta(z) &= S_\theta^+(z) + (S_\theta^+(z^{-1}))^T \\ &= R_\theta(0) + \mathcal{A}(p)(zI - \mathcal{A}(p))^{-1}R_\theta(0) \\ &\quad + R_\theta(0)(z^{-1}I - \mathcal{A}(p))^{-T}\mathcal{A}(p)^T. \end{aligned}$$

Therefore, we have

$$\begin{aligned} &(zI - \mathcal{A}(p))S_\theta(z)(z^{-1}I - \mathcal{A}(p))^T \\ &= (zI - \mathcal{A}(p))R_\theta(0)(z^{-1}I - \mathcal{A}(p))^T \\ &\quad + (zI - \mathcal{A}(p))\mathcal{A}(p)(zI - \mathcal{A}(p))^{-1}R_\theta(0)(z^{-1}I - \mathcal{A}(p))^T \\ &\quad + (zI - \mathcal{A}(p))R_\theta(0)(z^{-1}I - \mathcal{A}(p))^{-T}\mathcal{A}(p)^T(z^{-1}I - \mathcal{A}(p))^T \\ &= (zI - \mathcal{A}(p))R_\theta(0)(z^{-1}I - \mathcal{A}(p))^T \\ &\quad + \mathcal{A}(p)R_\theta(0)(z^{-1}I - \mathcal{A}(p))^T + (zI - \mathcal{A}(p))R_\theta(0)\mathcal{A}(p)^T, \end{aligned}$$

since $(zI - \mathcal{A}(p))\mathcal{A}(p)(zI - \mathcal{A}(p))^{-1} = \mathcal{A}(p)$. Thus,

$$\begin{aligned} &\mathcal{F}^{-1}(z)S_\theta(z)\mathcal{F}^{-T}(z^{-1}) \\ &= (zR_\theta(0) - \mathcal{A}(p)R_\theta(0))(z^{-1}I - \mathcal{A}(p))^T + z^{-1}\mathcal{A}(p)R_\theta(0) \\ &\quad - \mathcal{A}(p)R_\theta(0)\mathcal{A}(p)^T + zR_\theta(0)\mathcal{A}(p)^T - \mathcal{A}(p)R_\theta(0)\mathcal{A}(p)^T \\ &= R_\theta(0) - z^{-1}\mathcal{A}(p)R_\theta(0) - zR_\theta(0)\mathcal{A}(p)^T \\ &\quad + \mathcal{A}(p)R_\theta(0)\mathcal{A}(p)^T + z^{-1}\mathcal{A}(p)R_\theta(0) - \mathcal{A}(p)R_\theta(0)\mathcal{A}(p)^T \\ &\quad + zR_\theta(0)\mathcal{A}(p)^T - \mathcal{A}(p)R_\theta(0)\mathcal{A}(p)^T \\ &= R_\theta(0) - \mathcal{A}(p)R_\theta(0)\mathcal{A}(p)^T, \end{aligned}$$

and (5) establishes (4). \square